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Department of Electrical and Electronics Engineering

Subject Name: Electrical Machines - I **Subject Code: EE T33**

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UNIT I
MAGNETIC CIRCUITS AND ELECTRO MECHANICAL ENERGY CONVERSION

Simple magnetic circuit calculations– B-H Relationship – Magnetically induced emf and force – AC operation of magnetic circuits – Hysteresis and Eddy current losses - Energy in magnetic system – Field energy and mechanical force – Energy conversion via electric field

Two Marks

1. **What is magnetic circuit?**
   The closed path followed by magnetic flux is called magnetic circuit
2. **Define magnetic flux?**
   The magnetic lines of force produced by a magnet is called magnetic flux it is denoted as Φ and its unit is Weber
3. **Define magnetic flux density?** April 2014
   It is the flux per unit area at right angles to the flux it is denoted by B and unit is Weber/m^2
4. **Define magneto motive force?**
   MMF is the cause for producing flux in a magnetic circuit. the amount of flux setup in the core decent upon current(I)and number of turns(N).the product of NI is called MMF and it determine the amount of flux setup in the magnetic circuit MMF=NI ampere turns (AT)
5. **Define reluctance?**
   The opposition that the magnetic circuit offers to flux is called reluctance. It is defind as the ratio of MMF to flux. It is denoted by S and its unit is AT/m
6. **Define magnetic flux intensity?**
   It is defined as the mmf per unit length of the magnetic flux path. it is denoted as H and its unit is AT/m H=NI/L
7. **Define permeability?**
   Permeability of a material means its conductivity for magnetic flux. Greater the permeability of material, the greaters its conductivity for magnetic flux and vice versa.
8. **Define relative permeability?**
   It is equal to the ratio of flux density produced in that material to the flux density produced in air by the same magnetizing force.

   $$\mu r=\mu/\mu 0$$

9. **State the principle of electromechanical energy conversion?**
The mechanical energy is converted in to electrical energy which takes place through either by magnetic field or electric field

10. **Distinguish between statically induced emf and dynamically induced emf?**
When emf induced in a conductor is stationary in a magnetic field then we call it statically induced emf.If emf is induced in a conductor due to relative motion between conductor and the field then it call it as dynamically induced emf.

11. **Give example for single and multiple excited systems?**
Single excited system-reluctance motor, single phase transformer, relay coil Multiply excited system-alternator, electro mechanical transducer

12. **Why do all practical energy conversion devices make use of the magnetic field as a coupling medium rather than electric field?**
When compared to electric field energy can be easily stored and retrieved form a magnetic system with reduced losses comparatively. Hence most all practical energy conversion devices make use of magnetic medium as coupling

13. **Write the application of single and doubly fed magnetic systems?**
Singly excited systems are employed for motion through a limited distance or rotation through a prescribed angle Whereas multiply excited systems are used where continues energy conversion take place and in ease of transducer where one coil when energized the care of setting up of flux and the other coil when energized produces a proportional signal either electrical or mechanical.

14. **Why energy stored in a magnetic material always occur in air gap**
In iron core or steel core the saturation and aging effects form hindrance to storage. Built in air gap as reluctance as well permeability is constant, the energy storage takes place linearly without any complexity. Hence energy is stored in air gap in a magnetic medium

15. **What is the significance of co energy?** April 2015, April 2012
When electrical energy is fed to coil not the whole energy is stored as magnetic energy .the co energy gives a measure of other energy conversion which takes place in coil then magnetic energy storage
    1. Field energy
    2. Coenergy

16. **Write the expression for the mechanical energy output when the armature moves from one position to other with constant coil current?**
Let us assume armature moves from position xa to xb for a constant coil current
The mechanical energy is,

$$\Delta Wm = \int_{xa}^{xb} Ff dx = \Delta Wf$$

17. **What are the losses in a transformer?**
In a transformer, there exist two types of losses.
(i). The core gets subjected to an alternating flux, causing core losses.
(ii). The winding carry currents when transformer is loaded, causing copper losses.
(i). Core (or) Iron losses
Due to alternating flux set up in the magnetic core of the transformer, it undergoes cycle of magnetization and demagnetization. Due to hysteresis effect there is loss of energy in this process which is called hysteresis loss.

$$\text{Hysteresis loss} = K_h B_m^{1.67} fv \text{ watts}$$

Where, K_h = Hysteresis constant depends on material

B_m = Maximum flux density

f = Frequency

v = Volume of the core

The induced e.m.f in the core tries to set up eddy currents in the core and hence responsible for the eddy current losses.

$$\text{Eddy current loss} = K_e B_m^2 f^2 t^2 \text{ watts / unit volume}$$

Where, K_e = Eddy current constant

t = Thickness of the core

The iron losses are minimized by using high grade core material like silicon steel having very low hysteresis loss and by manufacturing the core in the form of laminations.

(ii). Copper losses

The copper losses are due to the power wasted in the form of I^2R loss due to the resistances of the primary and secondary windings. The copper loss depends on the magnitude of the currents flowing through the windings.

$$\text{Total copper loss} = I_1^2 R_{1e} = I_2^2 R_{2e}$$

18. Write the energy balance equation for motor? Nov'2012

Mechanical energy o/p-=electrical energy i/p-increase in field energy

$$Ff\,dx = id\lambda - dWf.$$

19. Define self-inductance?

The property of a coil that opposes any change in the amount of current flowing through it is called self inductance

20. Define mutual inductance?

The property of a coil to produce emf in a coil due to change in the value of current or flux in it is called mutual inductance.

20. State ampere law **April 2012**

The magnetic field created by an electric current is proportional to the size of that electric current with a constant of proportionality equal to the permeability of free space.

$$B \equiv \frac{\mu_o I}{2\Pi r}$$

Where B magnetic field in Tesla

μ_o Permeability of free space

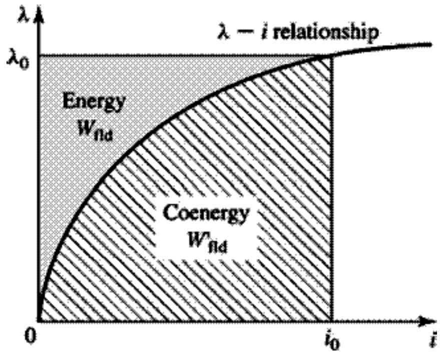
I – Current in the wire A

R – Radius from the wire

21. What is called rotating magnetic fiels? **April 2015**

A magnetic field having constant amplitude but whose axis continuously rotates in a plane with a certain speed is called rotating magnetic field.

22. Draw the i-λ curve of magnetic system for fixed position of armature and represent field energy and coenergy. April 2015



23. Write the factors on which hystresis loss depend April 2015

Hysteresis loss depends on the following factors

1. The hysteresis loss is directly proportional to the area under the hysteresis curve i.e. area of the hysteresis loop.
2. It is directly proportional to frequency i.e. number of cycles of magnetization per second.
3. It is directly proportional to volume of the material. It can be shown that quantitatively the hysteresis loss in joules per unit volume of the material in one cycle is equal to the area of the hysteresis loop

24. Define hystresis loss in ferromagnetic material April 2014

When a magnetic material is subjected to repeated cycles of magnetization and demagnetization, it results into disturbance in the alignment of the various domain. Now energy gets stored when magnetic field is established and energy is returned when field collapses. But due to hysteresis, all the energy is never returned though field completely collapses. This loss of energy appears as heat in the magnetic material. This is called as hysteresis loss.

25. Define the term electrical port, mechanical port and coupling field Nov' 2013

26. Write the basic equation of electro mechanical energy conversion Nov'2013

the energy balance equation becomes

(Wei - Ohmic losses) = (Wmo+Wms+Mechanical energy losses)+ (Wes+Coupling field losses).

Welec = Wmech + Wfld.

27. Define energy density April 2014

Energy density is the amount of energy stored in a given system or region of space per unit volume or mass, though the latter is more accurately termed specific energy

Deriv tions from other qu ntities = E/V

I unit J/m3

28. State faraday law April 2014

Whenever a flux linking in the coil changes emf always induced in the conductor the magnitude of induced emf is proportion l to r te of ch nge flux link ge e =NdΦ/dt

29. Write expression for force / torque in a system with phase transformer. **April 2013**

30. What is energy density in electric field? **April 2013**

Energy density per unit volume = $\int_{D1}^{D2} E_c \, dD_c$ v/m^3

31. What is a multiple excited magnetic field system? Give example. **Nov 2012**

In special devices more than one excitation coils are necessary. Such systems are called multiple excited systems. Very commonly used multiple excited systems use two excitation coils and are called doubly excited systems. The examples are synchronous motors, alternators, d.c. shunt machines, loudspeakers etc. in which separate excitation coils are provided on stator and rotor.

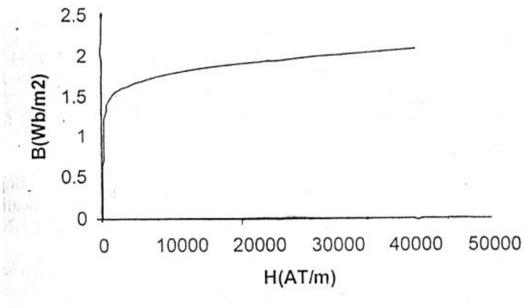
32. How do you define coenergy? **April 2015, April 2012**

The area underneath the magnetization curve (ψ - i) is known as coenergy and that can be defined by

$$W_t^1(i, x) = \int_0^i \psi(i, x) di$$

33. What is mean by B-H curve?

A magnetic hysteresis loop. A curve that expresses the magnetization of a magnetic material when placing the magnetic material such as iron and ferrite into a coil and changing the magnetic field to apply, in a graph is called B-H curve.



11 Marks

1. Write short notes on magnetic circuits

Introduction to Magnetic Circuits

All of us are familiar with a magnet. It is a piece of solid body which possesses a property of attracting iron pieces and pieces of some other metals. This is called a natural magnet . While as per the discovery of Scientist Oersted we can have an electromagnet. Scientist oersted stated that any current carrying conductor is always surrounded by a magnetic field. The property of such current is called magnetic effect of an electric current. Natural magnet or an electromagnet, both have close relation with electromotive force (e.m.f.), mechanical force experienced by conductor, electric current etc. To understand this relationship it is necessary to study the fundamental concepts of magnetic circuits.

Laws of Magnetism:

There an two fundamental laws of magnetism which are as follows :

L w 1 It st tes th t „like m gnetic poles repel nd unlike poles attract each other

L w 2 This l w is experiment lly proved by scientist coulomb nd hence known s coulomb‟s law.

The force (F) exerted by one pole on the other pole is,

a) Directly proportional to the product of the pole strengths.

b) Inversely proportional to the square of the distance between them, and

c) Nature of medium surrounding the poles.

Mathematically this Law can be expressed as,

F

Where,

M_1 and M_2 are pole strengths of the poles while d is distance between the poles.

$$F = \frac{K M_1 M_2}{d^2}$$

Where,

K depends on the nature of the surroundings and called permeability.

Magnetic Field:

We have seen that magnet has its influence on the surrounding medium. *The region around a magnet within which the influence of the magnet can be experienced is called magnetic field. Existence of such field can be experienced with the help of compass needle iron or pieces of metals or by bringing another magnet in vicinity of a magnet.

Magnetic Lines of Force:

The magnetic field of magnet is represented by imaginary lines around it which are called Magnetic Lines of Force. Note that these lines have no physical existence, these are purely imaginary and were introduced by Michael Faraday to get the visualization of distribution of such lines of force.

$\propto \frac{M_1 M_2}{d^2}$

2. Obtain the expression for B-H relationship. Also draw the B-H curve and explain its concept.
Nov' 2014

B-H CURVE or Magnetization curve

The magnetic field strength H is $\frac{NI}{l}$. As current in coil change, magnetic field strength also changes. Due to this flux produced and hence the flux density also changes. So there exists a particular relationship between B and H for a material
which can be shown on the graph.

Key Point: The graph between the flux density (B) and the magnetic field strength the magnetic material is called as its magnetization curve or B-H curve.

Let us obtain the B-H curve experimentally for a magnetic material. The arrangement required is shown in the Fig. 1.1.

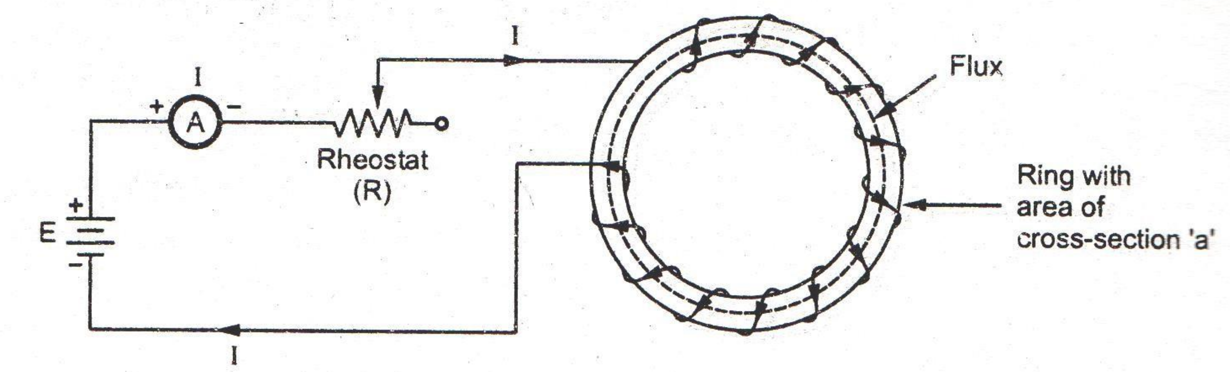


Fig. 1.1

The ring specimen as a mean length of 'l' metres with a cross-sectional are of „ " square metres. Coil is wound for 'N' turns carrying a current 'I' which can be varied by changing the variable resistance 'R' connected in series. Ammeter is connected to measure the current. For measurement of flux produced, fluxmeter can be used which is not shown in the Fig. 1.1.

So H can be calculated as while B can be calculated as $\frac{NI}{l}$ while B can be calculated as $\frac{\Phi}{a}$ for various values of current and plotted.

With the help of resistance R, I can be changed from zero to maximum possible value.

The B-H curve takes the following form, as shown in the Fig. 1.2

The graph can be analysed as below :

I. Initi l portion Ne r the origin for low v lues of „H', the flux density does not r pidly. This is represented by curve OA. The point A is called as instep.

II. Middle portion : In this portion as 'H' increases, the flux density B increases. This is almost straight line curve. At point 'C' it starts bending again, The C where this portion bends is called as knee point.

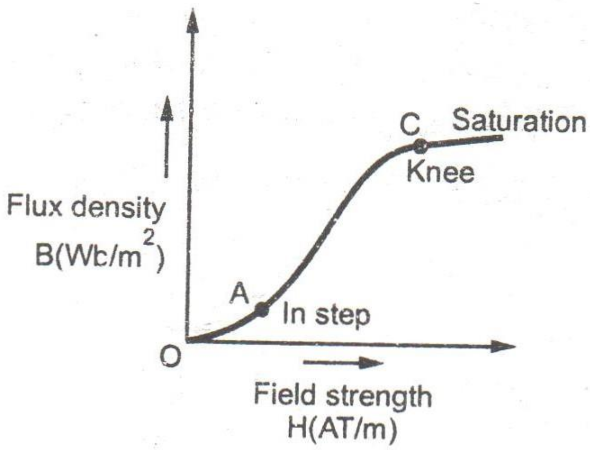


Fig. 1.2. B-H curve

III. Saturation portion : After the knee point, rate of increase in 'B' reduces drastically. Finally the curve becomes parallel to 'X' axis indicating that any increase in 'H' hereafter is not going to cause any change in „B‟. The ring is s id to be s tur ted nd region s s tur ted region.

According to molecular theory of magnetism, when all molecular magnets align themselves in the same direction due to application of H, saturation occurs.

B-H CURVE and Permeability

From B-H curve, a curve of relative permeability , and H can also be obtained.

As B = μH

$= \mu_o \mu_r H$

$$\mu_r = \frac{B}{\mu_o H}$$

is 4π X 10^{-7}. So B/H is nothing but slope of B-H curve.

Key point: So the slope of B-H curve at various points decide the value of relative permeability at that point.

Initially the slope is low so value of ,is also low. At knee point, the value of slope of B-H curve is maximum and hence , is maximum. But in saturation region the value of maximum and hence is maximum. But in saturation region the value of falls down to very low value as slope of B-H curve in saturation region is almost zero.

The curve of , against H is shown in the Fig. 1.3 for a magnetic material.

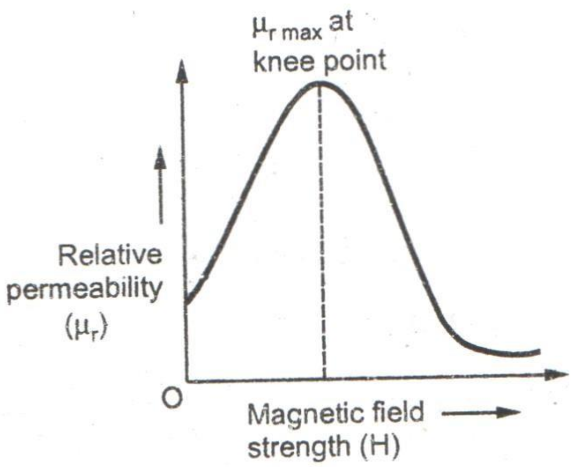


Fig. 1.3 - H curve for magnetic material

μ_r

μ_o

μ_r

μ_r μ_r μ_r

μ_r

μ_r

μ_r

The value of is 1 which is constant in case of non magnetic material. The slope of B-H curve is constant i.e. it is a straight line passing through the origin as shown in Fig. 1.4

Fig. 1.4 - H curve for non magnetic material

Practical use of B-H Curves

While designing the magnetic circuits, magnetization curves are useful to design the values of B corresponding to H, From this, proper material with required relative permeability can be selected. The various materials like iron, steel are generally represented by the B-H curves and -H curves.

Magnetic hysteresis

Instead of plotting B-H curve only for increase in current if plotted for one complete cycle of magnetization (increase in current) and demagnetization (decrease in current) then it is called hysteresis curve or hysteresis loop.

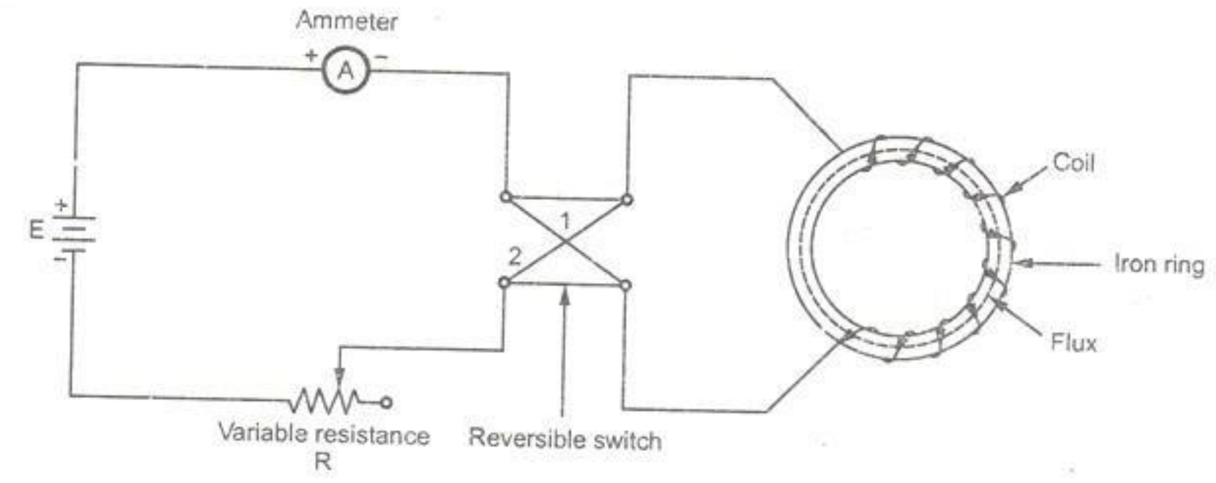


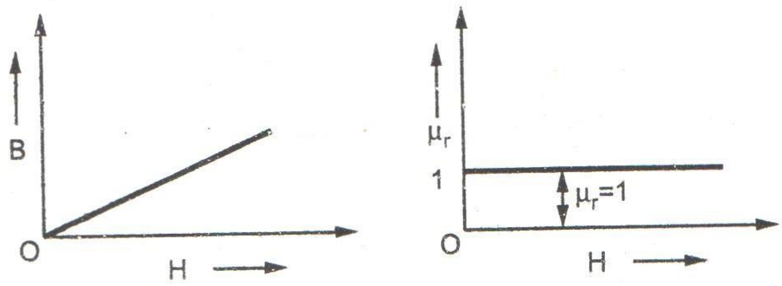
Fig. 1.5 Experimental set up to obtain hysteresis loop

Consider circuit consisting of b ttery „E‟, n mmeter, v ri ble resist nce R nd reversible switch shown in the Fig. 1.5.

Steps in Obtaining Hysteresis Loop

i. Initially variable resistance is kept maximum so current through the circuit is very low. The field strength H = $\frac{NI}{l}$ is also very low. So as current is increased, for low values of field strengths, flux density do not increase rapidly. But after the knee density increases rapidly upto certain point. This point is called point of saturation. Thereafter any change in current do not have an effect on the flux density. This curve is nothing but the magnetization curve (B-H curve) . This is the initial part of hysteresis loop.
ii. After the saturation point, now current is again reduced to zero. Due to this field also reduces to zero. But it is observed that flux density do not trace the curve back but falls back as compared to previous magnetization curve. This phenomenon of falling back of flux density while demagnetization cycle is

μ_r



μ_r

μ_r

called hysteresis. Hence due to this effect, when current becomes exactly zero, there remains some magnetism associated with a coil and hence the flux density. The core does not get completely demagnetized though current through coil becomes zero.This value of flux density when exciting current through the coil and magnetic field strength is reduced to zero is called residual flux density or remanent flux density. This is also called residual magnetism of the core. The magnitude of this residual flux or magnetism depends on the nature of the material of the core. And this property of the material is called retentivity.

iii. But now if it is required to demagnetize the core entirely then it is necessary to reverse the direction of the current through the coil. This is possible with the help of the intermediate switch.

Key Point: The value of magnetic field strength required to wipe out the residual flux denstiy is called the coercive force. It is measured interms of coercivity.

iv. If now this reversed current is increased, core will get saturated but in opposite direction. At this point flux density is maximum but with opposite direction.

v. If this current is reduced to zero, again core shows a hysteresis property and does not get fully demagnetized. It shows same value of residual magnetism but with opposite direction.

vi. If current is reversed again, then for a certain magnitude of field strength complete demagnetization of the core is possible.

vii. And if it is increased further, then saturation in the original direction is achieved completing one cycle of magnetization and demagnetization.

The curve plotted for such one cycle turns out to be a closed loop which is called hysteresis loop. its nature is shown in the Fig. 1.6.

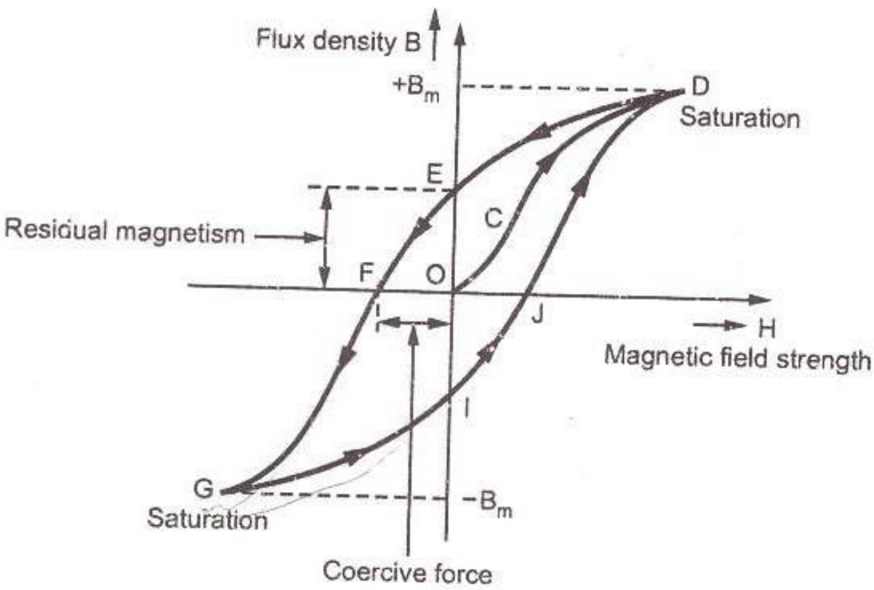


Fig. 1.6. Hysteresis loop

Part of Curve Represents What ?

i. **O-C-D :** Region corresponding to norm l m gnetiz tion curve incre sed form „o" to 'Im x' corresponding to 'Bm„. M ximum flux density is + Bm.

ii. **D-E :** Current reduced to zero, but core cannot be completely demagnetized O-E represents residual magnetism and residual flux density, denoted by $+B_r$.

iii. **E-F:** Current is reversed and increased in reversed direction to get complete demagnetization of the core. O-F represent coercive force required to completely wipe out + Br.

iv. **F-G:** Current is increased in reversed direction till saturation in opposite direction is achieved. Maximum flux density same but with opposite direction i.e. - Bm.

v. **G-I:** Current is reduced to zero but again flux density lags and core cannot be completely demagnetized. O-I represents residual flux density in other direction ie. $-B_r$.

vi. **I-J:** Current is again reversed and increased till complete demagnetization is achieved.

vii. **J-D:** Current is again increased in original direction till saturation is reached. Corresponding flux density is again + Bm.

Theory Behind Hysteresis Effect

As seen from the loop 'O-C-D-E-F-G-I-I-D' shown in the Fig. A.6, the flux density B always lags behind the values of magnetic field strength H. When H is zero, corresponding flux density is + B,. This effect is known as hysteresis.

This can be explained with the help of theory of molecular magnets inside a material. When a ferromagnetic material is subjected to a magnetic field strength, all the molecular magnets inside, align themselves in the direction of the applied m.m.f. If this applied force is removed or reduced some of the molecular magnets remain in the aligned state and magnetic neutralization of the material is not fully possible. So it continues to show some magnetic properties which is defined as the residual magnetism.

The value of the residual magnetism as said earlier depends on the quality of the magnetic material and the treatment it receives at the time of manufacturing. This property is called as retentivity.

Key Point: Higher the value of retentivity, higher the value of the power of the magnetic material to retain its magnetism. For high retentivity, higher is the coercive force required.

It can be measured in terms coercivity of the material.

Hysteresis Loss

According to the molecular theory of magnetism groups of molecules acts like magnets, which are magnetized to saturation. This magnetism is developed because of the magnetic effect of electron spins, which are known as 'domains'.

When the material is unmagnetized, the axis of the different domains are in various direction.

Thus the resultant 'magnetic effect is zero. When the external magnetomotive force is applied the axes of the various domains are oriented. The axes coincide with the direction of the magnetomotive force. Hence the resultant of individual magnetic effects is a strong magnetic field.

When a magnetic material is subjected to repeated cycles of magnetization and demagnetization, it results into disturbance in the alignment of the various domain. Now energy gets stored when magnetic field is established and energy is returned when field collapses. But due to hysteresis, all the energy is never returned though field completely collapses. This loss of energy appears as heat in the magnetic material. This is called as hysteresis loss. So disturbance in the alignment of the various domains causes hysteresis loss to take place. This hysteresis loss is undesirable and may cause undesirable high temperature rise due to heat produced. Due to such loss overall efficiency also reduces.

Such hysteresis loss depends on the following factors

1. The hysteresis loss is directly proportional to the area under the hysteresis curve i.e. area of the hysteresis loop.
2. It is directly proportional to frequency i.e. number of cycles of magnetization per second.
3. It is directly proportional to volume of the material. It can be shown that quantitatively the hysteresis loss in joules per unit volume of the material in one cycle is equal to the area of the hysteresis loop.

Magnetic Loss [Core Loss or iron Loss]

The magnetic losses can be classified as

i) Hysteresis loss

ii) Eddy current loss

The magnetic loss occurs in the core hence they are known as core losses. Since material is generally iron or its alloy this loss is also referred as iron loss. The magnetic loss will result in the following effects,

i) It reduces the efficiency of the electrical equipment.

ii) It increases the temperature because of heating of the core.

Key Point: The hysteresis loss can be reduced by selecting good quality magnetic material.

The area of hysteresis loop should be narrow. Silicon steel is employed for the core material so that hysteresis loss can be minimized.

Key Point: The eddy current loss can be reduced by using thin laminations for the core.

Faradays Laws of Electromagnetic Induction

Electromagnetic induction produces potential difference (voltage) across a conductor when it is exposed to a varying magnetic field. In 1831, Michael Faraday, an English physicist gave one of the most basic law of electromagnetism called **Faraday's Law of electromagnetic induction**

This law explains the working principle of most of the electrical motors, generators, electrical transformers and inductors. This law shows the relationship between electric circuit and magnetic field . Faraday law proved that emf is induced in the coil when flux linked with it changes.

First Law

Whenever the number of magnetic lines of force (flux) linking with a coil or circuit changes, an e.m.f gets induced in that coil or circuit.

Second Law

The magnitude of the induced emf is directly proportional to the rate of change of flux Linkage (Flux X Turns of coil).

Flux linkages = Flux X Number of turns of coil

Now as per the first law, e.m.f. will get induced in the coil and as per second law the magnitude of e.m.f. is proportional to the rate of change of flux linkages.

$$e = N\frac{d\varnothing}{dt}$$

Now s per Lenz‟‟s l w, the induced e.m.f. sets up a current in such a direction so as to oppose very cause producing it. Mathematically this opposition is expressed by a negative sign.

Thus such an induced e.m.f. is mathematically expressed alongwith its sign as,

$$e = N\frac{d\varnothing}{dt} \text{ volts}$$

The total flux linkages of the coil are given by,

$$= N\varnothing \text{ (WbT)}$$

Where N is the number of turns of the coil.

Hence induced e.m.f, can be expressed as,

$$e = \frac{d\lambda}{dt}$$

The negative sign indicates that induced e.m.f, opposes the changes in the flux

linkages, according to Lenz's law.

Lenz's law

This rule is based on the principles derived by German physicist Heinrich Lenz.

The Lenz‟‟s l w st ted th t, „the direction of n induced e.m.f produced by the electrom gnetic induction is such that it sets up a current which always opposes the cause that is responsible for inducing the e.m.f.

Lenz law states that whenever an emf is induced, the induced current is in such a direction so as to oppose the cause producing it .

In short the induced e.m.f always opposes the cause producing it, which is represented by a negative sign, mathematically in its expression.

$$E = -N\frac{d\varnothing}{dt}$$

Fleming's right hand rule (used to find the direction of current)

This rule is used for generators , it shows the direction of induced current when a conductor moves in a magnetic field.

The right hand is held with the thumb, first finger and second finger mutually perpendicular to each other (at right angles), as shown in the Fig.1.7

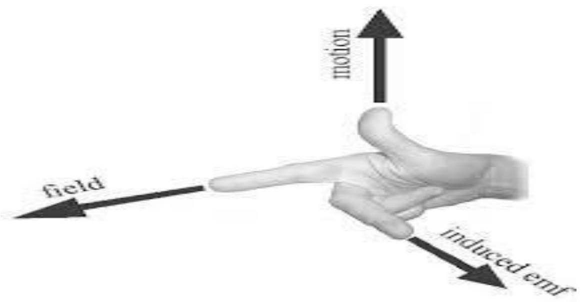


Fig 1.7 Fleming Right Hand Rule

- The thumb represents the direction of motion of the conductor.
- The first finger represents the direction of the field.
- The second finger represents the direction of the induced or generated current.

Fleming's left hand rule (used to find the direction of force)

Fleming"s left h nd rule is for electric motors nd it shows the direction of the thrust on conductor carrying a current in a magnetic field. As shown in the fig 1.8.

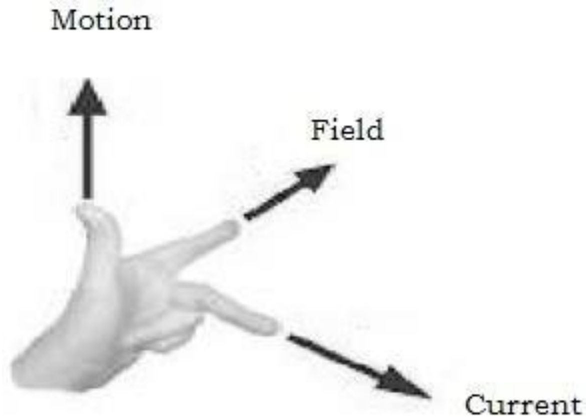


Fig 1.8 Fleming Left Hand Rule

- The left hand is held with the thumb, first finger and second finger mutually at right angles.
- The first finger represents the direction of the magnetic field.
- The second finger represents the direction of the current.
- The thumb represents the direction of the resultant motion.

Tips for remembering Fleming's left and right hand rule

- In lph bets A,B,C,..L,M,…Z. The letters L and M area besides each other and hence left hand rule (L) is for MOTOR (M); the remaining is obviously the right hand rule and generator, so right hand rule (R) is for generator (G).

3. Explain dynamically and statically induced emf with examples. April 2015

Nature of the Induced E.M.F.

E.M.F. gets induced in a conductor, whenever there exists change in flux with that conductor, according to F r d y"s L w. uch ch nge in flux c n be brought bout by different methods.

Depending upon the nature of methods, the induced e.m.f. is classified as,

1) Dynamically induced e.m.f, and

2) Statically induced e.m.f.

Dynamically Induced E.M.F.

The change in the flux linking with a coil, conductor or circuit can be brought about by its motion relative to magnetic field. This is possible by moving flux with respect to coil conductor or circuit or it is possible by moving conductor, coil, circuit with respect to stationary magnetic flux.

Key Point: Such an induced emf which is due to physical movement of coil, conductor with respect to flux or movement of magnet with respect to stationary coil, conductor is called dynamically induced emf or motional induced emf.

Statically induced E.M.F.

Key Point: The change in flux lines with respect to coil can be achieved without physically moving the coil or the magnet. Such induced e.m.f in a coil which is without physical movement of coil or a magnet is called statically induced e.mf

Explanation : To have an induced e.m.f. there must be change in flux associated with a coil. Such a change in flux can be achieved without any physical movement by increasing and decreasing the current producing the flux rapidly, with time.

Consider an electromagnet which is producing the necessary flux for producing e.m.f. Now let current through the coil of an electromagnet be an alternating one. Such alternating current means it changes its magnitude periodically with time. This produces the flux which is also alternating i.e. changing with time. Thus there exists $d\Phi/dt$ associated with coil placed in the viscinity of an electromagnet. This is responsible for producing an e.m.f. in the coil. This is called statically induced e.m.f.

Key Point: It can be noted that there is no physical movement of magnet or conductor,
is the alternating supply which is responsible for such an induced e.mf

The concept of statically induced e.m.f. is shown in the Fig. 1.9.

Such an induced e.m.f. can be observed in case of a device known as transformer.

Note : Due to alternating flux linking with the coil itself, the e.m.f. gets induced in that coil itself which carries an alternating current.

The statically induced e.m.f. is further classified as,

1) Self induced e.m.f. and 2) Mutually induced e.m.f.

We shall study now these two types of statically induced e.m.f.s.

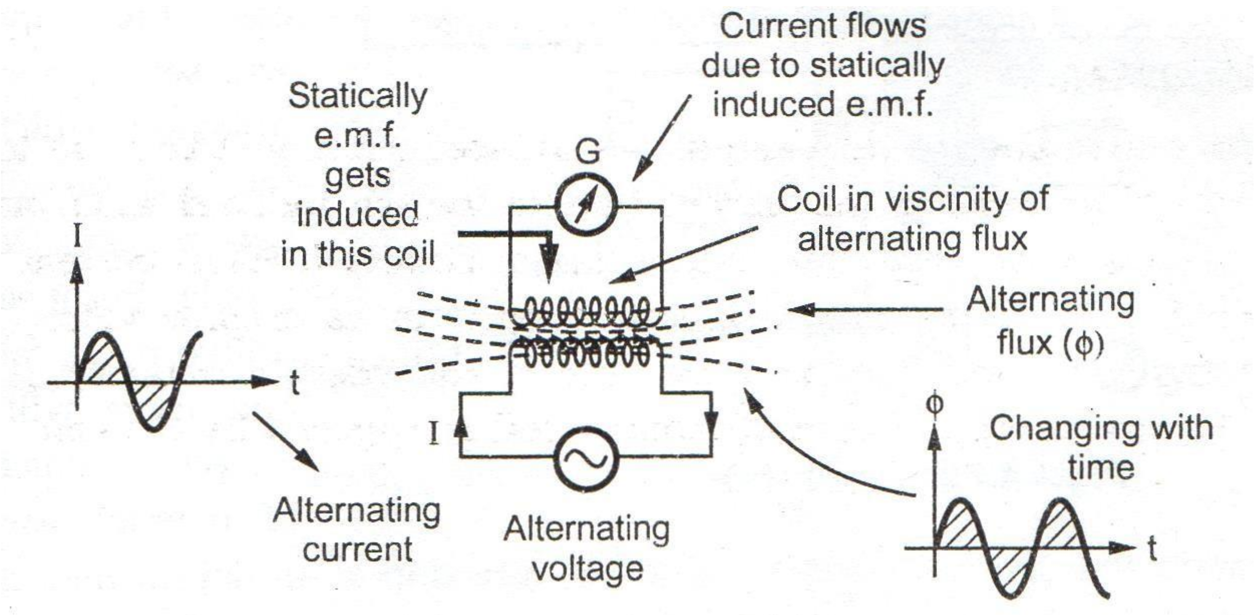


Fig. 1.9. Concept of statically induced e.m.f

Self Induced E.M.F

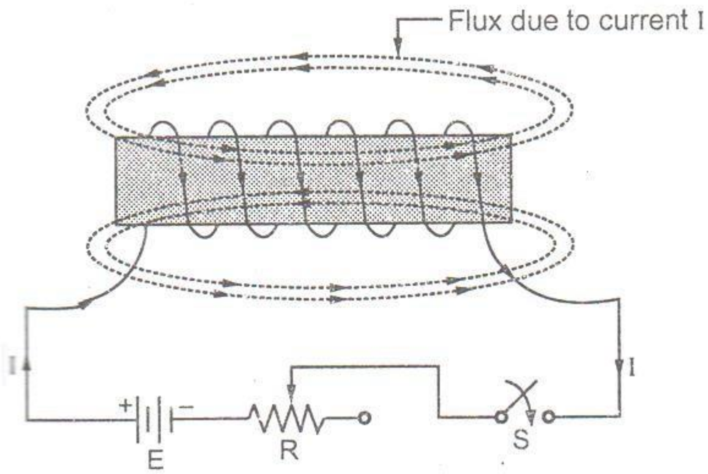


Fig. 1.10.

Consider a coil having 'N' turns and carrying current 'I' when switch 'S' is in closed position. The current magnitude can be varied with the help of variable resistance connected in series with battery, coil and switch as shown in the Fig. 1.10.

The flux produced by the coil links with the coil itself. The total flux linkages of coil will be N Φ Wb-turns. Now if the current 'I' is changed with the help of variable resistance, then flux produced will also change, due to which flux linkages will also change.

Hence according to Faraday's law, due to rate of change of flux linkages there will be induced e.m.f. in the coil. So Without physically moving coil or flux there is induced e.m.f. in the coil. The phenomenon is called self induction.

The e.m.f. induced in a coil due to the change of its own flux linked with it is called self induced e.m.f.

Key Point: The self induced e.mf lasts till the current in the coil is changing. The direction of such induced e.mf can be obtained by Lenz's law.

Mutually Induced E.M.F.

If the flux produced by one coil is getting linked with another coil and due to change in this flux produced by first coil, there is induced e.m.f. in the second coil, then such an e.m.f. is called mutually induced e.m.f.

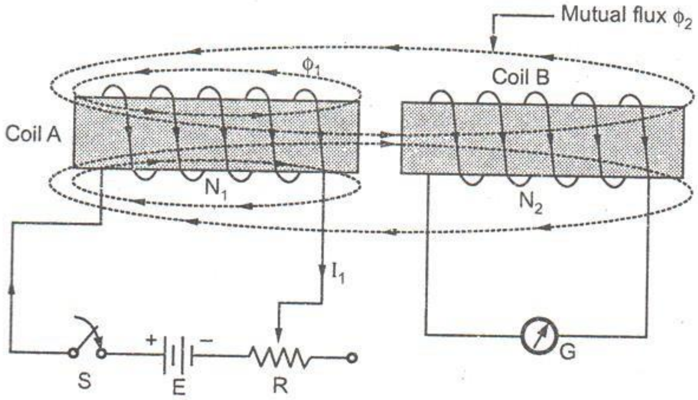


Fig. 1.11. Mutually Induced E.M.F.

Consider two coils which are placed adjacent to each other as shown in the Fig. 1.10. The coil A has N_1 turns while coil B has N_2 number of turns. The coil A has switch S, variable resistance R and battery of 'E' volts in series with it. A galvanometer c is connected across coil B to sense induced e.m.f. and current because of it. Current through coil A is I_1 producing flux Φ_1. Part of this flux will link with coil B i.e. it will complete its path through coil B as shown in the Fig. 1.11. This is the mutual flux.

Now if current through coil A is changed by means of variable resist nce R, then flux Φ_1 changes. Due to this, flux ssoci ted with coil B, which is mutu l flux Φ_2 also changes. Due to Faraday's law there will be induced e.m.f. in coil B which will set up a current through coil B, which will be detected by galvanometer G.

Key Point: Any change in current through coil A produces e.m.f in coil B, this phenomenon is called mutual induction and e.m.f is called mutually induced e.m.f.

4. Derive the expression for the energy stored and energy density in a magnetic field. April 2012

A.C operation of Magnetic Circuits:

In many applications and machines such as transformers and a.c. machines, the magnetic circuits are excited by a.c. supply. In such an operation, inductance plays vital even in steady state operation though in d.c. it acts as a short circuit. In such a case. The flux is determined by the a.c. voltage applied and the frequency. Thus the exciting current has to adjust itself according to the flux so that every time B-H relationship is satisfied.

Consider a coil having N turns wound on iron core as shown in the Fig.1.13 The coil carries an ltern ting current I v rying sinusoid lly. Thus the flux Φ produced by the exciting current i is lso sinusoidally varying with time.

$$\therefore \Phi = \Phi_m \sin \omega t \qquad .....(1)$$

Where,

Φ_m = Maximum value of flux in core

ω = 2πf where f is frequency in Hz

Fig. 1.13

According to faraday's law, as flux changes with respect to coil, the e.m.f. gets reduced in the coil given by

$$e = N \frac{d\phi}{dt} = N \frac{d}{dt}[\quad \sin \omega t]$$

$$\therefore \quad e = N \Phi_m \omega \cos \omega t \quad \text{volt} \qquad .....(2)$$

$$= \text{Maximum value} = N\Phi_m \omega$$

$$E = \text{r.m.s value} = \frac{E_m}{\sqrt{2}} = \frac{N\Phi_m \omega}{\sqrt{2}}$$

$$E = \frac{N\Phi_m 2\pi f}{\sqrt{2}} = 4.44 \text{ f N} \qquad .....(3)$$

But, $\Phi_m = A_c B_m$

Where, A_c = Area of cross-section of core.

B_m = Maximum value of flux density in Wb/ m^2

The sign of the e.m.f induced must be determined according to Lenz's law, opposing the change in the flux. The current and flux are in phase as current produces flux instantaneously. Now induced e.m.f. is cosine term and thus leads the flux and current by 90°. This is called back e.m.f. as it opposes the applied voltage. The resistance drop is small and is neglected in most of the electromagnetic devices.

Energy Stored under A.C. Operation

The instantaneous electric power input into the magnetic circuit through the coil terminals is given by,

$$P = e\,I \quad \text{but } e = \frac{d\lambda}{dt}$$

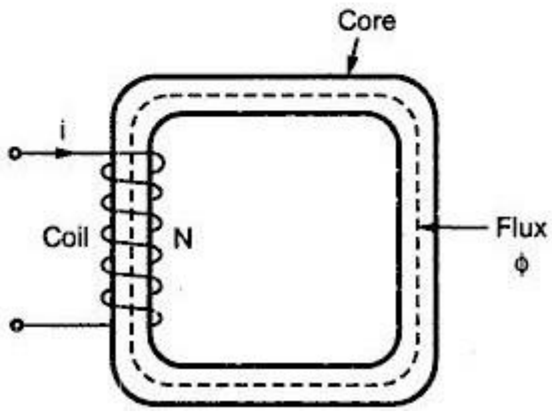
$$P = i\frac{d\lambda}{dt} \qquad .....(4)$$

The power is rate of change of energy hence energy input which gets stored in the magnetic field during the interval $t_1$to t_1 is,

$$W_f \int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} i\frac{d\lambda}{dt}\,dt$$

$$W_f = \int_{\lambda_1}^{\lambda_2} i\,d\lambda \qquad .....(5)$$

Thus W_f is the increase in the field energy as the flux linkages of the coil change



E_m

from $\lambda 1$ nd $\lambda 2$ during the interv l t_1 to t_2.

Now, $H_c l_c$ = m m f = N i

.◦. $i = \frac{H_c l_c}{N}$ …..(6)

Using in the equation (5),

$$W_f = \int_{\lambda_1}^{\lambda_2} \frac{H_c l_c}{N} d\lambda$$

But $\lambda = N\Phi$ nd $\Phi = B_c a_c$ i.e. $\lambda = N B_c a_c$

The flux density of the core is changing from B1 to B2 as the flux linkages change from $\lambda 1$ to $\lambda 2$.

$$W_f = \int_{B_1}^{B_2} (\frac{H_c l_c}{N}) N a_c dB_c$$

$$W_f = a_c l_c \int_{B_1}^{B_2} H_c \, dB_c$$

This is the field energy in terms of field quantities

Now $a_c l_c$ = Volume of the core

.◦. Per unit volume = $\int_{B_1}^{B_2} H_c \, dB_c$ J/

This is called field energy density.

Energy In Magnetic System

Energy can be stored or retrieved from a magnetic system by means of an exciting coil connected to an electric source. Consider, for example the magnetic system of an attracted armature relay. The resistance of the coil is shown by a series lumping outside the coil which then is regarded as an ideal loss less coil current causes magnetic flux to be established in the magnetic circuit. It is assumed that all the flux confined to the iron core and therefore links all the N turns creating the coil flux linkages of

$\lambda = N\Phi$ …..(1)

The flux linkage causes a reaction emf of

$e = \frac{d\lambda}{dt}$ …..(2)

to ppe r t the coil termin ls with pol rity (s per lenz‟s l w) shown in the Fig. The ssociated circuit equation is

v=iR + e

$= iR + \frac{d\lambda}{dt}$ …..(3)

The electric energy input into the ideal coil due to the flow of current I in time dt is

dW_e = ei dt …..(4)

Assuming for the time being that the armature is held fixed at position x, all the input energy is stored in the magnetic field. Thus

$d W_e$ = ei dt = $d W_f$ …..(5)

Where dW_f is the change in field energy in time dt. When the expression for e in Eq. (2) is substituted in Eq(5),we have

$d W_e$ = ei dt = F dΦ = $d W_f$ …..(6)

Where F = Ni, the magneto motive force (mmf).

The re1ationship i-λ or F-λ is function l one corresponding to the m gnetic circuit which in general is non linear (and is also history-dependent, i.e, it exhibits hysteresis). The energy absorbed by the field for finite change in flux linkages for flux is obtained from Eq. (4.6) as

m^3

$$\Delta W_f = \int_{\lambda_1}^{\lambda_2} i(\lambda)\,d\lambda = \int_{\Phi_1}^{\Phi_2} F(\Phi)\,d\Phi \quad .....(7)$$

As the flux in the magnetic circuit undergoes a cycle Φ_1 to Φ_2 to Φ_1, an irrecoverable loss in energy takes place due to hysteresis and eddy-currents in the iron, assuming here that these losses are separated out and are supplied directly by the electric source. This assumption renders the ideal coil and the magnetic circuit as a conservative system with energy interchange between themselves so that the net energy is conserved.

The energy bsorbed by the m gnetic system to est blish flux Φ (flux link ges λ) from initi l zero flux is

$$W_f = \int_0^{\lambda} i(\lambda)\,d\lambda = \int_0^{\Phi} F(\Phi)\,d\Phi \quad .....(8)$$

This then is the energy of the magnetic field with given mechanical configuration when its state corresponds to flux Φ (flux link ges λ).

Magnitude of magnetically induced e.m.f

Consider a conductor of length l metres moving in the air gap between the poles of the magnet.
If plane of the motion of the conductor is parallel to the plane of the magnetic field then there is no cutting of flux lines and there can not be any induced e.m.f in the conductor such condition is shown in the Fig.
Key point : When plane of the flux is parallel to the plane of the motion of conductors then there is no cutting of flux, hence no induced emf.

Key Point: When plane of the flux is perpendicular
cutting of flux is maximum and hence induced emf is also

Fig.1.15

Consider a conductor moving with velocity v m/s such that its plane of motion or direction of velocity is perpendicular to the direction of flux lines as shown in Fig. 1.15 (a).

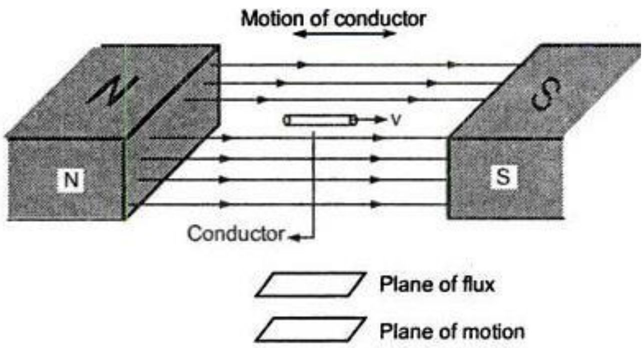
B = Flux density in Wb/

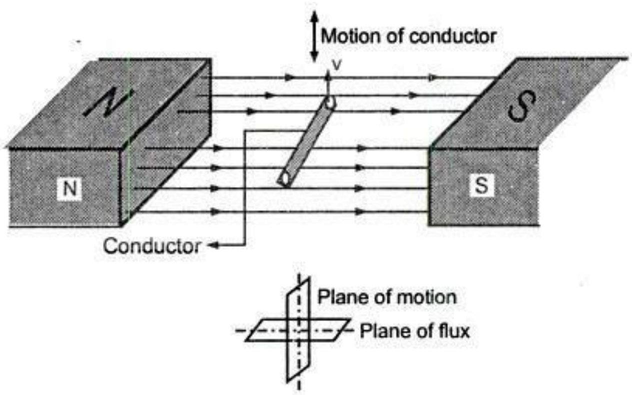
I = Active length of conductor in metres.

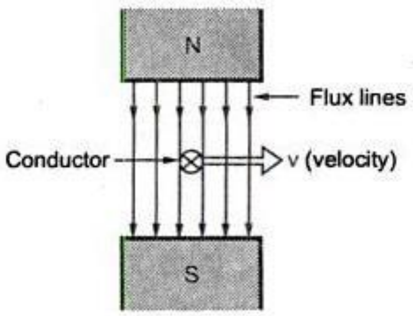
(This is the length of conductor which is actually responsible for cutting of flux lines)

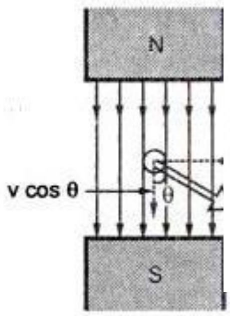
v = Velocity in m/sec.

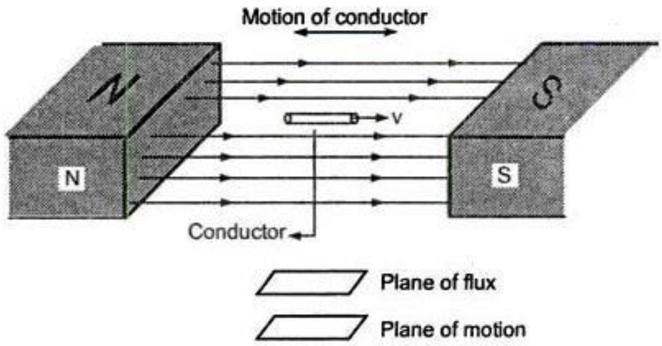
Let this conductor is moved through distance dx in a small time interval dt, then


Motion of conductor
N
S
N
S
v
Conductor
Plane of flux
Plane of motion


Motion of conductor
v
N
S
N
S
Conductor
Plane of motion
Plane of flux


N
Flux lines
Conductor
v (velocity)
S


N
v cos θ
θ
S


Motion of conductor
N
S
N
S
v
Conductor
Plane of flux
Plane of motion

m^2

Area swept by conductor = l × dx m^2

Flux cut by conductor = Flux density x Area swept

$d\Phi = B \times l \times dx$ Wb

According to faraday's law, magnitude of induced e.m.f. is proportional to the rate of change of flux.

$\therefore e = \frac{\text{Flux cut}}{\text{time}}$

$= \frac{d\Phi}{dt}$ [Here N = 1 as single conductor]

$= \frac{B\, l\, dx}{dt}$

But, $\frac{dx}{dt}$ = Rate of change of displacement

= Velocity of the conductor

= v

e = B l v in volts

This is the induced e.m.f. when plane of motion is exactly perpendicular to the plane of flux. This is maximum possible e.m.f. as plane of motion is at right angles to plane of the flux.

But if conductor is moving with a velocity v but at a certain angle Θ measured with respect to direction of the field (plane of the flux) as shown in the Fig.1.15 (b) then component of velocity which is v sin θ is perpendicular to the direction of flux and hence responsible for the induced e.m.f..The other component v cos θ is parallel to the plane of the flux and hence will not contribute to the dynamically induced e,m.f. Under this condition magnitude of induced e.m.f. is given by,

e = B l v sin θ in volts

where,

θ is measured with respect to plane of the flux.

Force on a Current Carrying Conductor in a Magnetic Field

The magnetic effects of electric current are very useful in various practical applications like generators motors etc. One of such important effect is force experienced by a current carrying conductor in a magnetic field.

Let a straight conductor, carrying a current is placed in a magnetic field as shown in fig.1.16 (a).

The magnetic field in which it is placed has a flux pattern as shown in the fig.1.16(a).

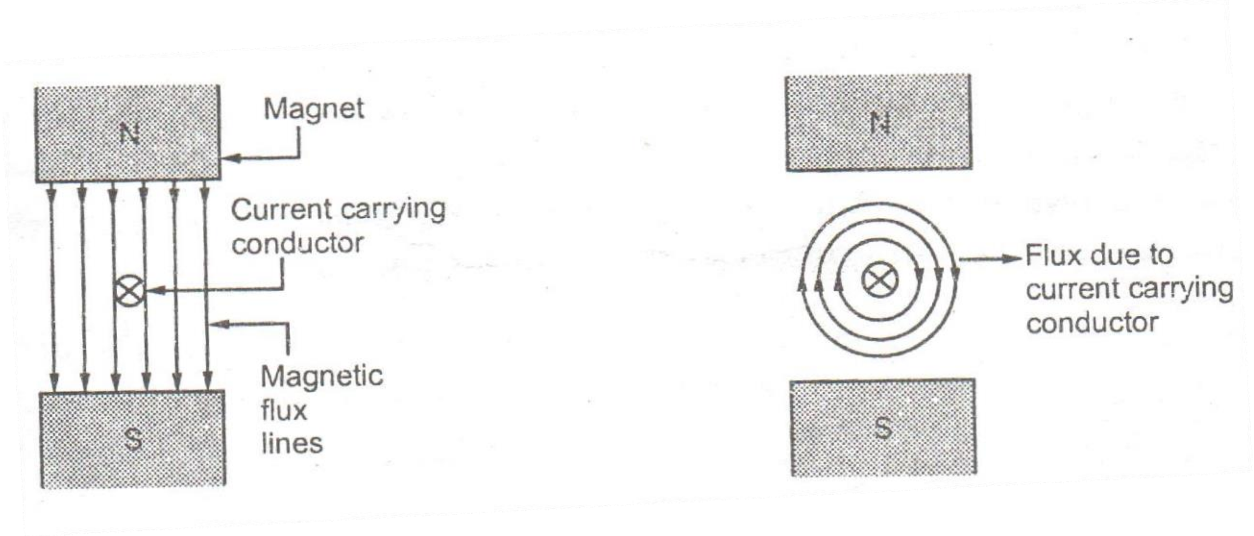


Fig. 1.16 Current carrying conductors in a magnetic field

(a) Flux due to magnet (b) Flux due to current carrying conductor

Now current carrying conductor also produces its own magnetic field around it.

Assuming current direction away from observer i.e. into the paper, the direction of its flux can be determined by right hand thumb rule. This is clockwise as shown in the

Fig. 1.16 (b). [For simplicity, flux only due to current carrying conductor is shown in the Fig. 1.16 (b).]
Now there is presence of two magnetic fields namely due to permanent magnet and due to current carrying conductor. These two tluxes interact with each other. Such interaction is shown in the Fig. 1.17 (a).
This interaction as seen is in such a way that on one side of the conductor the two lines help each other, while on other side the two try to cancel each other. This means on left hand side of the conductor shown in the Fig. 1.17 the two fluxes are in the same direction and hence assisting each other. As against this, on the right hand side of the conductor the two fluxes are in opposite direction hence trying to cancel each other. Due to such interaction on one side of the conductor, there is accumulation of flux lines (gathering of the flux lines) while on the other side there is weakening of the flux lines.
The resultant flux pattern around the conductor is shown in the Fig. 1.34 (b).

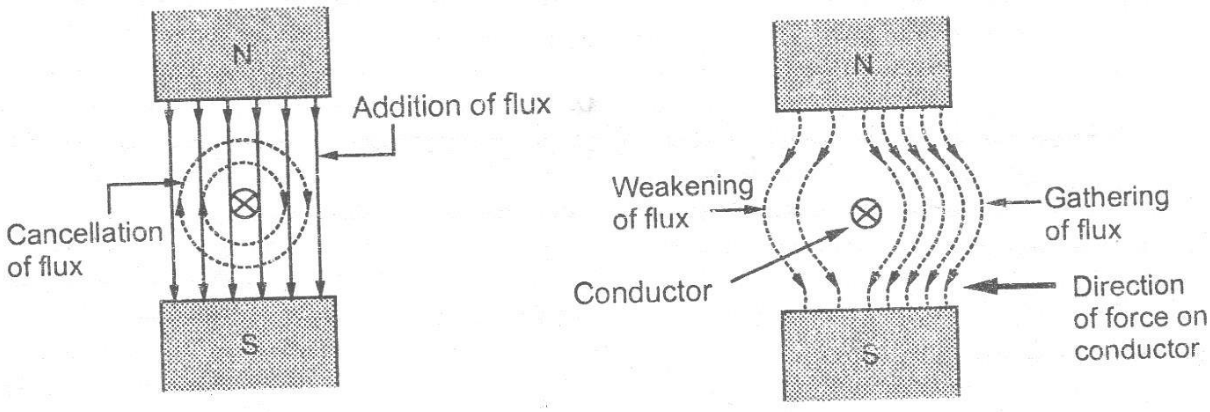


Fig. 1.17 Interaction of the two flux lines

According to properties of the flux lines, these flux lines will try to shorten themselves. While doing so, flux lines which are gathered will exert force on the conductor. So conductor experiences a mechanical force from high flux lines area towards low flux lines area i.e. from left to right for a conductor shown in the Fig. 1.17

Key Point: Thus we can conclude that current carrying conductor placed in the magnetic field, experiences a mechanical force, due to interaction of two fluxes.

This is the basic principle on which D.C. electric motors work and hence also called motoring action.

Magnitude of Force Experienced by the Conductor

The magnitude of the force experienced by the conductor depends on the following factors,

1) Flux density (B) of the magnetic field in which the conductor is placed measured in Wb/m^2 i.e. Tesla.

2) Magnitude of the current I passing through the conductor in Amperes.

3) Active length 'l' of the conductor in metres.

The active length of the conductor is that part of the conductor which is actually under the influence of magnetic field.

5. **Derive the expression for force and torque of a single excited magnetic field system in terms of energy and coenergy. April 2015**
6. **For singly excited magnetic field system, derive the relation for the magnetic stored energy in (a) current excited system (b) voltage controlled system. April 2015**
7. **Explain the single excited and multiply excited magnetic field system with neat sketch. April 2012**
8. **Explain the field energy and mechanical force developed in an attracted armature relay excited by an electric source. April 2013**
9. **Explain the terms energy and coenergy. How is force between two parallel phases in a singly excited system calculated? Nov'2014**

ENERGY IN MAGNETIC SYSTEM

The analysis of singly excited magnetic system includes the derivations of expressions of electrical input energy, magnetically stored field energy and the mechanical force. Consider attracted type electromechanical armature relay as a singly excited magnetic system. It is shown in the Fig. 1.18

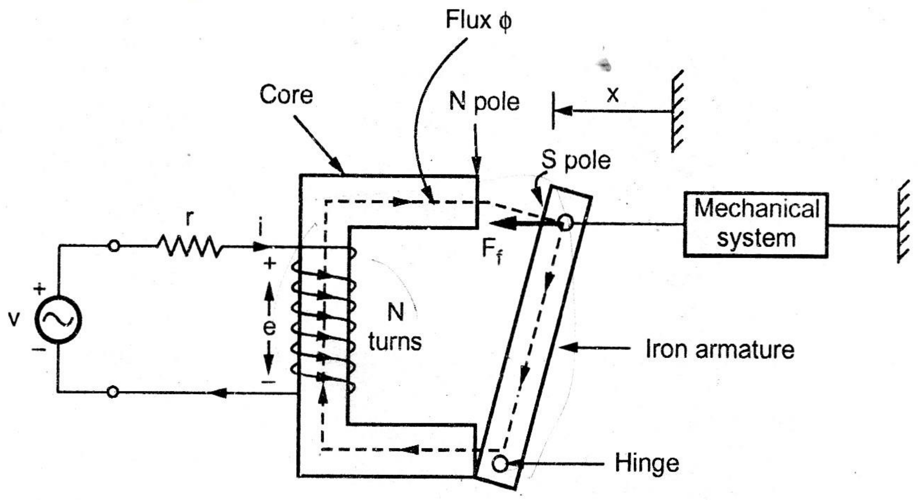


Fig. 1.18 Singly excited attracted armature relay

The following **assumptions** are made while performing the analysis of singly excited magnetic system,

1. The resistance of the exciting coil is assumed to be present in lumped form, outside the coil. Thus coil is lossless and ideal.
2. The leakage flux does not take part in energy conversion process so it is neglected as practically it is small. Hence all the flux is confined to the iron core and links all the N turns of coil.

   Thus if total flux is ϕ and turns of coil N then,

$$\lambda = \text{Flux linkages} = N\phi \quad \text{Wb-turns} \qquad \dots (1)$$

3. The leakage inductance is negligible.
4. There is no energy loss in the magnetic core. The reluctance of the iron path is neglected as negligible.

Electric Energy Input

Due to the flux linkages λ, the reaction e.m.f. e exists, whose direction is so as to oppose the cause producing it i.e. voltage v according to Lenz's law as shown in the Fig. 1.18 This e.m.f. is given by,

$$e = \frac{d\lambda}{dt} \quad \text{...(2)}$$

Applying Kirchhoff's voltage law to the coil circuit,

$$v = ir + e \quad \text{... (3 (a))}$$

$$\therefore \quad v = ir + \frac{d\lambda}{dt} \quad \text{...(3 (b))}$$

Field Energy and Mechanical Force

Consider that the magnetic field produces a mechanical force F_f as shown in the relay. This force drives the mechanical system consisting active and passive mechanical elements.

Let the armature moves a distance dx in positive direction i.e. in the direction of force. The mechanical work done by the magnetic field then can be obtained as,

$$dW_m = F_f \, dx \quad \text{...(1)}$$

As per the energy balance equation for electrical input,

Mechanical energy output dW_m = [Electrical energy input dW_e]- [Change in stored Energy dW_f]

$$\therefore \quad F_f \, dx = I \, d\lambda - dW_f \quad \text{... (2)}$$

In such electromech nic l systems the independent v ri bles c n be (i, x) or (λ,x).

Case 1: The independent variables are (I,x) i.e. current constant

Thus λ ch nges s i nd x hence,

$$\lambda = \lambda \, (i,x)$$

$$\therefore \quad d\lambda = \frac{\partial \lambda}{\partial i} di + \frac{\partial \lambda}{\partial x} dx \quad \text{...(3)}$$

While the stored field energy is

$$W_f = W_f \, (i,x)$$

$$d \quad = \frac{\partial w_f}{\partial i} di + \frac{\partial w_f}{\partial x}$$

10. State and explain with suitable examples of doubly excited magnetic field system. Also mention their applications. Nov' 2014

1.7 Multiple Excited Magnetic System

For continuous energy conversion devices like alternators, synchronous motors etc., multiply excited magnetic systems are used. In practice, doubly excited systems are very much in use.

The Fig. 1.14 shows double excited magnetic system. This system has two independent sources of excitations. One source is connected to coil on stator while other is connected to coil on rotor.

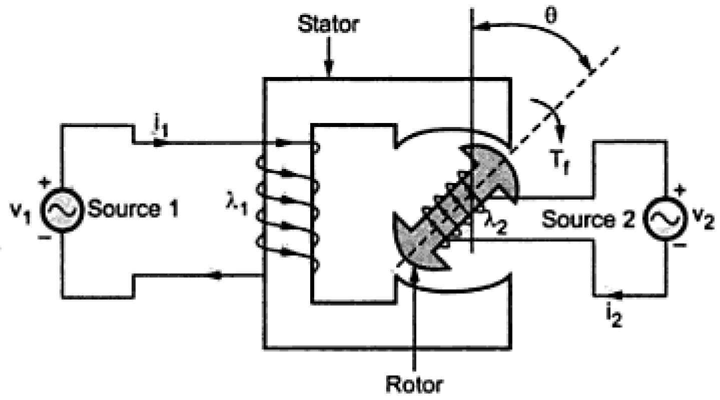


Fig. 1.14 Doubly excited system

Let i_1 = Current due to source 1

i_2 = Current due to source 2

λ_1 = Flux linkages due to i_1

λ_2 = Flux linkages due to i_2

θ = Angular displacement of rotor

T_f = Torque developed

Due to two sources, there are two sets of three independent variables i.e. $(\lambda_1, \lambda_2, \theta)$ or (i_1, i_2, θ).

Case 1 : Independent variables λ_1, λ_2, θ i.e. λ_1, λ_2 are constants

From the earlier analysis it is known,

$$T_f = \frac{-\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} \quad \text{... currents are variables (1)}$$

While the field energy is,

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 d\lambda_1 + \int_0^{\lambda_2} i_2 d\lambda_2 \quad ...(2)$$

Now let L_{11} = Self inductance of stator

L_{22} = Self inductance of rotor

$L_{12} = L_{21}$ = Mutual inductance between stator and rotor

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \quad ...(3)$$

and

$$\lambda_2 = L_{12} i_1 + L_{22} i_2 \quad ...(4)$$

Solve (3) and (4) to express i_1 and i_2 interms of λ_1 and λ_2 as λ_1 and λ_2 are independent variables.

Multiply (3) by L_{12} and (4) by L_{11},

$$L_{12}\,\lambda_1 = L_{11}\,L_{12}\,i_1 + L_{12}^2\,i_2$$

and $$L_{11}\,\lambda_2 = L_{11}\,L_{12}\,i_1 + L_{11}\,L_{22}\,i_2$$

Subtracting the two,

$$L_{12}\,\lambda_1 - L_{11}\,\lambda_2 = [L_{12}^2 - L_{11}\,L_{22}]\,i_2$$

$$i_2 = \left[\frac{L_{12}}{L_{12}^2 - L_{11}L_{22}}\right]\lambda_1 - \left[\frac{L_{11}}{L_{12}^2 - L_{11}L_{22}}\right]\lambda_2$$

$$\therefore \quad \boxed{i_2 = \beta_{12}\,\lambda_1 + \beta_{22}\,\lambda_2} \quad \ldots (5)$$

Note that negative sign is absorbed in definin, β.

Similarly i_1 can be expressed interms of λ_1 and λ_2 as,

$$\therefore \quad \boxed{i_1 = \beta_{11}\,\lambda_1 + \beta_{12}\,\lambda_2} \quad \ldots (6)$$

where

$$\beta_{11} = \frac{L_{22}}{L_{11}L_{22} - L_{12}^2}$$

$$\beta_{22} = \frac{L_{11}}{L_{11}L_{22} - L_{12}^2}$$

$$\beta_{21} = \beta_{12} = \frac{-L_{12}}{L_{11}L_{22} - L_{12}^2}$$

Using in equation (2),

$$W_f\,(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} [\beta_{11}\lambda_1 + \beta_{12}\lambda_2]\;d\lambda_1 + \int_0^{\lambda_2} [\beta_{12}\lambda_1 + \beta_{22}\lambda_2]\;d\lambda_2$$

Integrating the terms we get,

$$\therefore \quad \boxed{W_f\,(\lambda_1, \lambda_2, \theta) = \frac{1}{2}\beta_{11}\lambda_1^2 + 2\beta_{12}\lambda_1\lambda_2 + \frac{1}{2}\beta_{22}\lambda_2^2} \quad \ldots (7)$$

The self and mutual inductances of the coils are dependent on the angular position θ of the rotor.

Case 2 : Independent variables i_1, i_2, θ i.e. i_1 and i_2 are constants.

The torque developed can be expressed as,

$$T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta} \quad ...(8)$$

The co-energy is given by,

$$W_f'(i_1, i_2, \theta) = \int_0^{i_1} \lambda_1 di_1 + \int_0^{i_2} \lambda_2 di_2 \quad ...(9)$$

Using $\lambda_1 = L_{11} i_1 + L_{12} i_2$

and $\lambda_2 = L_{12} i_1 + L_{22} i_2$

$$W_f'(i_1, i_2, \theta) = \int_0^{i_1} [L_{11} i_1 + L_{12} i_2] di_1 + \int_0^{i_2} [L_{12} i_1 + L_{22} i_2] di_2$$

$$\therefore \quad \boxed{W_f'(i_1, i_2, \theta) = \frac{1}{2} L_{11} i_1^2 + 2 L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2} \quad ... (10)$$

Force in a Doubly Excited System :

$$F = \frac{\partial W_f'}{\partial \theta}(i_1, i_2, \theta)$$

where i_1 and i_2 are constants which are the stator and rotor current respectively.

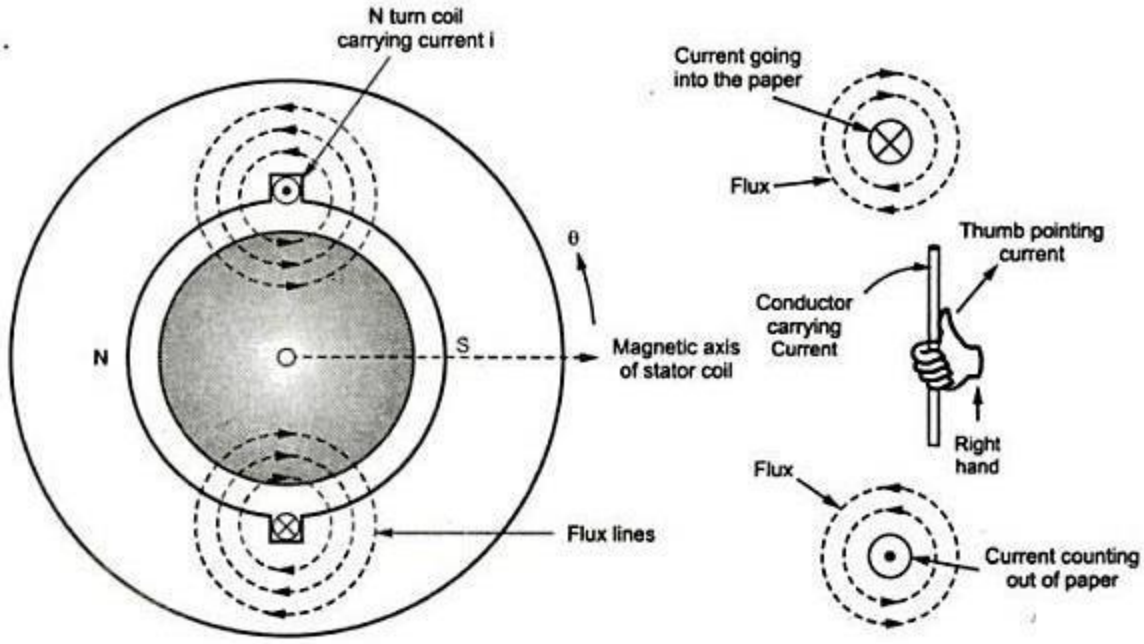
$$\therefore \quad F = \frac{\partial}{\partial \theta}\left[\frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2\right]$$

$$\therefore \quad \boxed{F = \frac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + i_1 i_2 \frac{\partial L_{12}}{\partial \theta} + \frac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta}}$$

11. With necessary diagram, discuss about MMF waves of distributed AC winding in details with its equations. April 2015

It is seen that the armature winding used for three phase alternators is distributed in nature. The efforts are made to place the coils in all the slots available per pole per phase. Such coils are then interconnected such that the magnetic field produced by armature after carrying current has same number of poles as that of field winding. Let us obtain the m.m.f space distribution of the current carrying armature by superimposing the m.m.f. space waves of individual coils.

Consider single multiturn coil, carrying N turns and is a **full pitch coil.** It carries a current i such that the total ampere turns produced by the coil is (Ni). The direction of current i is as shown in the Fig. 2.23 and hence direction of flux lines around a and a' can be obtained by using **right hand thumb rule.**



a) Coil producing NI ampere-turns b) Right hand thumb rule

The stator is wound for two poles same as the number of poles for which field winding is wound. Due to the flux lines produced by the coil, the North and South poles are induced on the stator periphery. The magnetic axis of the stator coil is from N pole to S pole as shown in the Fig. 2.23 (a).

The assumptions to obtain M.M.F. space wave are,

1. It is cylindrical rotor machine.
2. The armature and rotor are made up of high grade magnetic material hence permeability of these parts is much higher than air. Hence reluctance is low so entire reluctance can be assumed to be due to two air gaps.
3. Thus if total m.m.f. is Ni then half the m.m.f. is required to creat flux from rotor to stator in the air gap while half is required to create flux from stator to rotor in the air gap.

The flux lines radially crosses the air gap between rotor and stator twice, normal to the stator and rotor iron surfaces.

Consider a developed diagram of the coil shown in the Fig. 2.24 such that rotor surface is over the stator which is laid down flat.

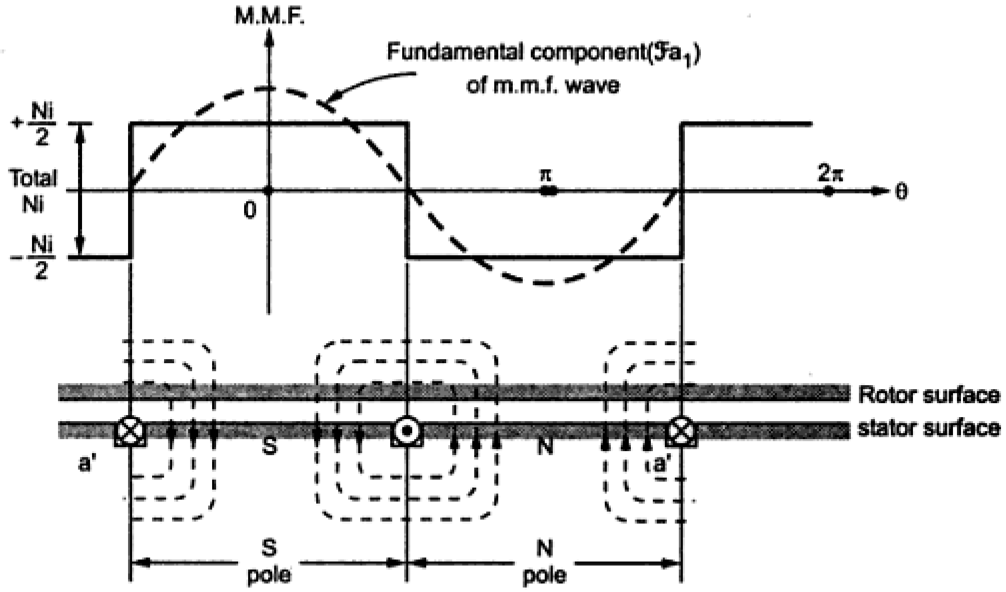


Fig. 2.24 M.M.F. space wave of a single coil

The m.m.f. and flux radially outwards from rotor to stator is assumed positive while that from stator to rotor is assumed negative. Thus m.m.f. distribution is stepwise giving a rectangular waveform. The m.m.f. $+\frac{Ni}{2}$ is used in setting flux from rotor to stator in the air gap. No m.m.f. required for iron path. Similarly the m.m.f. $-\frac{Ni}{2}$ is used in setting flux from stator to rotor in the air gap.

Thus m.m.f. changes suddenly from $-\frac{Ni}{2}$ to $+\frac{Ni}{2}$ at one slot while from $+\frac{Ni}{2}$ to $-\frac{Ni}{2}$ at other slot which is a pole pitch away. Total change in m.m.f. is abrupt and equal to Ni in crossing from one side to other of a coil.

M.M.F. change at any slot = Ni

The direction depends on the current direction.

The rectangular m.m.f. space wave can be resolved into its Fourier series which includes its fundamental component and a series of odd harmonics. The fundamental component of m.m.f wave is given by,

$$\mathfrak{F}_{a1} = \frac{4}{\pi}\frac{Ni}{2}\cos\theta = F_p\cos\theta \quad \dots (1)$$

Where θ = Electrical angle measured from stator magnetic axis.

Key Point: *This stator magnetic axis coincides with the positive peak of the m.m.f wave.*

$$\therefore \quad \mathfrak{F}_{a1}(\text{peak}) = F_p = \frac{4}{\pi}\frac{Ni}{2} \quad \dots (2)$$

The odd harmonics are to be neglected.

Key Point:*Equation (1) shows that the m.m.f. space wave is sinusoidal and when m.m.f. waves of individual distributed phase group coils add, the odd harmonics get cancelled.*

12. Write a short note about the rotating magnetic field in an electric machine. April 2013

2.9 Rotating Magnetic Field

It is seen that the m.m.f. wave produced by the single phase a.c. winding is pulsating, whose amplitude varies sinusoidally with time. This is because the current flowing in single phase a.c. winding is alternating in nature.

Now consider a magnetic field produced due to physical rotation of a permanent magnet in a space with a certain speed. It produces a magnetic field whose magnitude is constant but axis rotates in space with certain speed.

Key Point:*Thus a magnetic field having constant amplitude but whose axis continuously rotates in a plane with a certain speed is called* ***rotating magnetic field.***

In a pulsating m.m.f. wave, the axis is fixed while amplitude varies sinusoidally while in rotating magnetic field amplitude is constant but axis rotates with a certain speed.

Consider a **three phase a.c. winding**, either star or delta connected supplied from a three phase a.c. supply. This is shown in the Fig. 2.28.

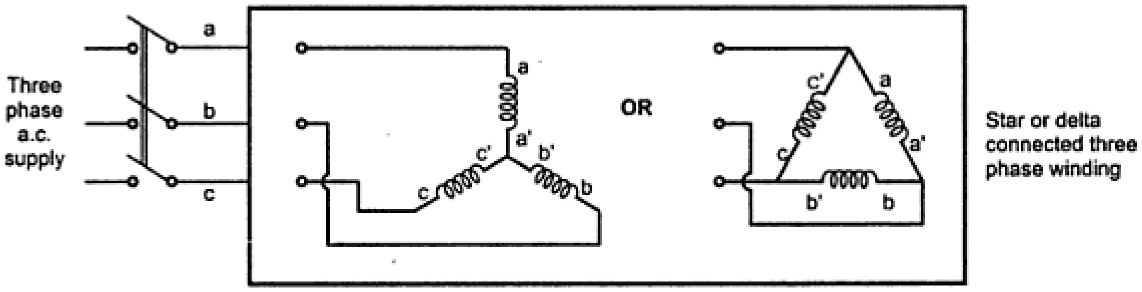


Fig. 2.28 Production of rotating magnetic field

The three sinusoidal phase currents flow simultaneously through the windings. These currents are displaced from each other by 120° electrical. These currents can be mathematically expressed as,

$$i_a = I_m \cos \omega t \quad \ldots(1a)$$

$$i_b = I_m \cos(\omega t - 120°) \quad \ldots(1b)$$

$$i_c = I_m \cos(\omega t - 240°) \quad \ldots(1c)$$

The phase sequence is assumed to be a-b-c.

As the supply is balanced and windings are identical, the maximum value I_m of each phase current is equal. These currents are shown in the Fig. 2.29.

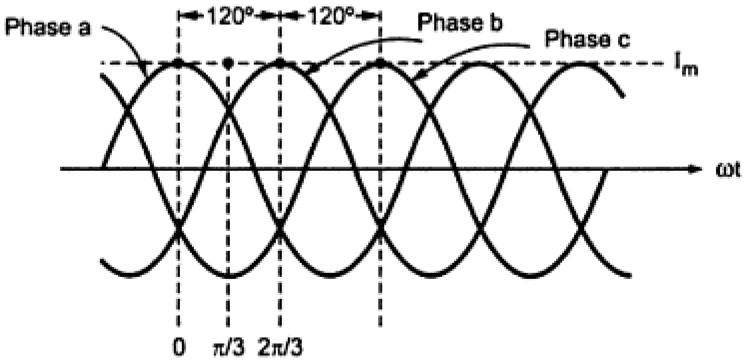


Fig. 2.29 Three phase currents

The instant is considered when phase **a** current has attained its maximum value. Such three currents produce three individual m.m.f. waves as discussed in the last section. These m.m.f. waves also displaced by 120° electrical from each other in space and their magnetic axes are coinciding with the axes of the respective phases as shown in the Fig. 2.30.

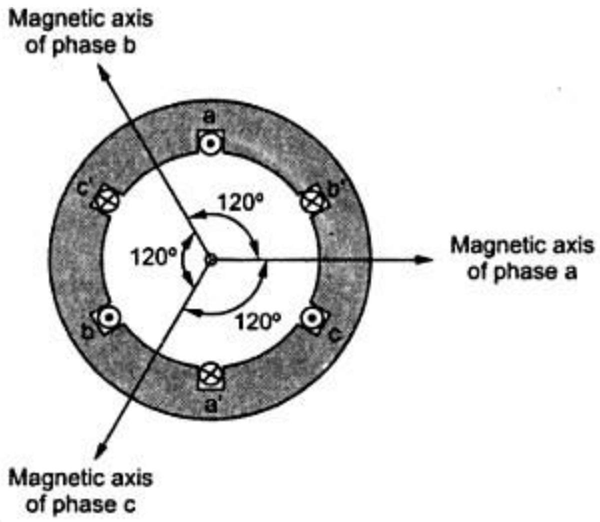


Fig. 2.30 Magnetic axes of three phases

As the three individual m.m.f. waves are separated by 120° electrical from each other in space, their mathematical representation can be obtained as,

$$\mathcal{F}_a = F_m \cos \omega t \cos \theta \qquad \text{...(2a)}$$

$$\mathcal{F}_b = F_m \cos(\omega t - 120°) \cos(\theta - 120°) \qquad \text{...(2b)}$$

$$\mathcal{F}_c = F_m \cos(\omega t - 240°) \cos(\theta - 240°) \qquad \text{...(2c)}$$

$$\text{where} \quad F_m = \frac{4}{\pi} K_c K_d \left[\frac{T_{ph}(\text{series})}{P}\right] I_m$$

From trignometric result,

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$$

$$\therefore \quad \mathcal{F}_a = \frac{1}{2}F_m \cos(\theta - \omega t) + \frac{1}{2}F_m \cos(\theta + \omega t) \qquad \text{...}\alpha = \theta \text{ and } \beta = \omega t$$

$$\therefore \quad \mathcal{F}_b = \frac{1}{2}F_m \cos(\theta - \omega t) + \frac{1}{2}F_m \cos(\theta + \omega t - 240°)$$

$$\therefore \quad \mathcal{F}_c = \frac{1}{2}F_m \cos(\theta - \omega t) + \frac{1}{2}F_m \cos(\theta + \omega t - 480°)$$

The resultant m.m.f. wave is the sum of the three individual pulsating m.m.f. waves.

$$\therefore \quad \mathcal{F}_R = \mathcal{F}_a + \mathcal{F}_b + \mathcal{F}_c$$

$$= \frac{3}{2}F_m \cos(\theta - \omega t) + \frac{1}{2}F_m \{\cos(\theta-\omega t) + \cos(\theta + \omega t - 240°) \; \cos(\theta + \omega t - 480°)\}$$

$$= F_1 + F_2 \qquad \text{...(3)}$$

$$\text{But} \quad \cos(\theta + \omega t - 480°) = \cos(\theta + \omega t - 360° - 120°) = \cos(\theta + \omega t - 120°)$$

$$\therefore \quad F_2 = \frac{1}{2}F_m\{\cos(\theta + \omega t) + \cos(\theta + \omega t - 240°) + \cos(\theta + \omega t - 120°)\}$$

$$\text{Now} \quad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\therefore \quad F_2 = \frac{1}{2}F_m\{\cos\theta\cos\omega t - \sin\theta\sin\omega t + \cos\theta\cos(\omega t - 240°)$$

$$-\sin\theta\sin(\omega t - 240°) + \cos\theta\cos(\omega t - 120°) - \sin\theta\sin(\omega t - 120°)\}$$

Now $\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \cos\beta$

$$\therefore F_2 = \frac{1}{2} F_m \{\cos\theta \cos\omega t - \sin\theta \sin\omega t + \cos\theta [\cos\omega t \cos(-240°) + \sin\omega t \sin(-240°)]$$

$$-\sin\theta[\sin\omega t \cos(-240°) - \cos\omega t \sin(-240°)] + \cos\theta [\cos\omega t \cos(-120°) + \sin\omega t \sin(-120°)]$$

$$-\sin\theta[\sin\omega t \cos(-120°) - \cos\omega t \sin(-120°)]\}$$

Now $\cos(-240°) = -0.5$, $\cos(-120°) = -0.5$, $\sin(-240°) = +0.866$, $\sin(-120°) = -0.866$

$$\therefore F_2 = \frac{1}{2} F_m \{\cos\theta \cos\omega t - \sin\theta \sin\omega t - 0.5\cos\theta\cos\omega t + 0.866\sin\omega t\cos\theta$$

$$+ 0.5 \sin\theta\sin\omega t + 0.866 \sin\theta\cos\omega t - 0.5\cos\theta \cos\omega t$$

$$-0.866\sin\omega t\cos\theta + 0.5\sin\theta\sin\omega t - 0.866\sin\theta\cos\omega t\}$$

$$\therefore F_2 = 0 \qquad \text{...(4)}$$

Using in (3),

$$\boxed{\mathcal{F}_R = \frac{3}{2} F_m \cos(\theta - \omega t)} \qquad \text{...(5)}$$

This equation shows that the resultant m.m.f. wave is a **travelling wave.** Its magnitude is constant equal to $\frac{3}{2}F_m$ while its phase angle changes linearily with time as ωt. So **its axis rotates in the air gap at a constant speed of ω rad(elect)/sec.**

Key Point : *This shows that when a three phase stationary windings are excited by balanced three phase a.c. supply then the resulting field produced is* ***rotating magnetic field.*** *Though nothing is physically rotating, the field produced is rotating in space having constant amplitude.*

Thus at $t_1 = 0$, $\mathcal{F}_R = \frac{3}{2} F_m \cos\theta$ when phase a is at its maximum. Thus axis of resultant m.m.f coincides with the axis of the phase **a**.

At $t_2 = \frac{2\pi}{3}$, $\mathcal{F}_R = \frac{3}{2} F_m \cos\left(\theta - \frac{2\pi}{3}\right)$ and phase b is at its maximum. Thus axis of resultant m.m.f. coincides with the axis of phase **b**.

At $t_3 = \frac{4\pi}{3}$, the axis of resultant m.m.f. coincides with the axis of phase **c** and so on.

Key Point : *So if phase sequence is a-b-c, the rotating magnetic field rotates in the direction of phase sequence.*

2.9.1 Speed of Rotating Magnetic Field

It is known that its speed is ω rad(elect)/sec. The mechanical angular velocity ω_m is related to electrical ω as,

$$\omega_m = \frac{2}{P}\omega$$

$$\therefore \quad \frac{2\pi N}{60} = \frac{2}{P}(2\pi f) \qquad \text{...As } \omega = 2\pi f \text{ electrical}$$

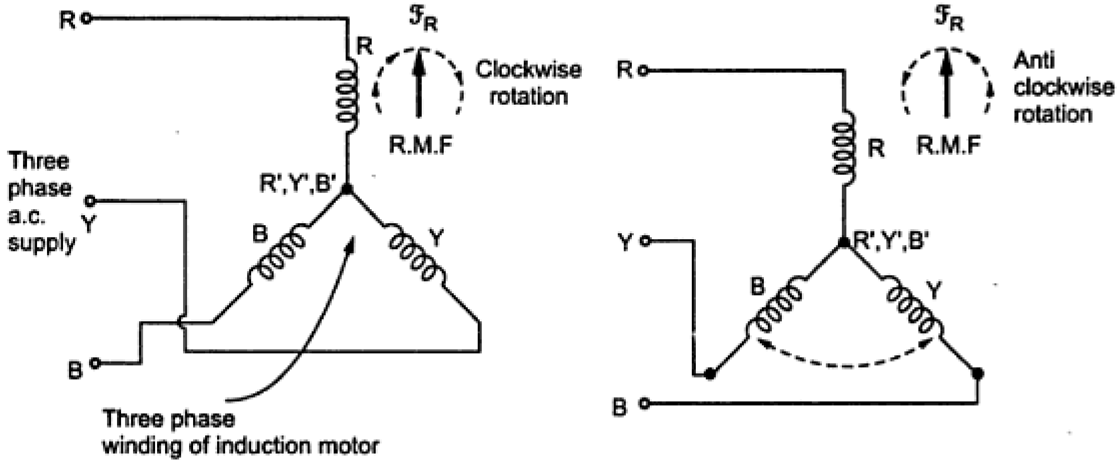
$$\therefore \quad \boxed{N = \frac{120 f}{P}}$$

This is the speed of the rotating magnetic field. This is called synchronous speed and denoted as N_s.

Key Point : *Thus rotating magnetic field rotates with synchronous speed N_s.*

2.9.2 Direction Rotating Magnetic Field

The direction of the R.M.F. is always from the axis of the leading phase of the three phase winding towards the lagging phase of the winding. In a phase sequence of R-Y-B, phase R leads Y by 120° and Y leads B by 120°. So R.M.F. rotates from axis of R to axis of Y and then to axis of B and so on. So its direction is clockwise as shown in the Fig.2.31(a). This direction can be reversed by interchanging any two terminals of the three phase windings while connecting to the three phase supply. The terminals Y and B are shown interchanged in the Fig. 2.31 (b). In such case the direction of R.M.F. will be anticlockwise.



(a) Clockwise rotation (b) Anticlockwise rotation

Fig. 2.31

As Y and B of windings are connected to B and Y from winding point of view the phase sequence becomes R-B-Y. Thus R.M.F. axis follows the direction from R to B to Y which is anticlockwise.

Key Point : *Thus by interchanging any two terminals of three phase winding while connecting it to three phase a.c. supply, direction of rotation of R.M.F. gets reversed.*

2.10 Torque in Round Rotor Machine

When current carrying conductor is placed in a magnetic field, the force is exerted on the conductor to cause the motoring action. This force is the basic cause of the production of torque. Practically current carrying conductor produces its own field while there exists the magnetic field in which it is kept. The two magnetic fields interact with each other and the torque is produced so as to align these two fields.

Key Point : *Thus for torque production, two magnetic fields must be present and interact with each other.*

In a.c. machines both stator and rotor carry currents and produce their own magnetic fields. These fields are shown in the Fig. 2.32.

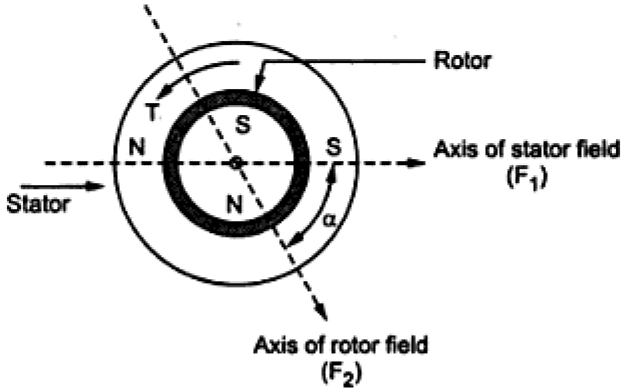


Fig. 2.32 Torque in round rotor machine

The stator and rotor are wound for 2 poles, producing north and south poles respectively on the stator and rotor surfaces. The direction of axis of each field is as shown in the Fig. 2.34. The angle between the two axes is α. Due to this angle, the torque appears so as to align the two axes.

Let F_1 = Peak value of sinusoidal m.m.f. stator wave

F_2 = Peak value of sinusoidal m.m.f. rotor wave

α = Angle between F_1 and F_2

To find the resultant of F_1 and F_2, these can be represented in the phasor form as shown in the Fig. 2.33. The angle between $\bar{F}_1$ and $\bar{F}_r$ is γ while the angle between $\bar{F}_2$ and $\bar{F}_r$ is δ

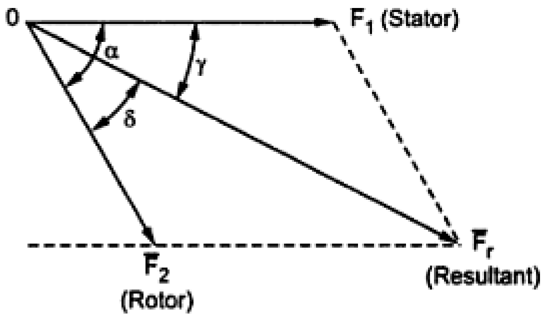


Fig. 2.33 Phasor sum of $\bar{F}_1$ and $\bar{F}_2$

The resultant $\bar{F}_r$ can be obtained using Law of parallelogram and is also sinusoidal m.m.f. space wave.

Using cosine rule, the peak value of resultant m.m.f. F_r is given by,

$$F_r^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\alpha \quad ...(2)$$

The field intensity H along the air gap is constant. The m.m.f. acorss the air gap is,

$$F_{air-gap} = H \times g$$

Where g = Radial length of air gap

This entire resultant m.m.f. is required to cross the flux across the air gap as reluctance of iron path is neglected.

$$\therefore \quad F_{air-gap} = F_r$$

$$= H_r g$$

Where H_r = Peak value of resultant field intensity

$$\therefore \quad H_r = \frac{F_r}{g} \quad ...(3)$$

The co-energy density of the co-energy stored in the air gap at a point is given by $(1/2)\mu_0 H^2$.

The average value of the co-energy density over the volume of air gap is $\mu_0/2$ times the average value of H^2.

$$\therefore \quad \text{Average value of co-energy density} = \frac{1}{2}\mu_0 \text{ (Average value of } H^2) \quad ...(4)$$

But H is purely sinusoidal hence,

$$\text{Average value of } H^2 = \frac{1}{2} \times [\text{Peak value of H}]^2$$

$$= \frac{1}{2}H_r^2 \quad ...(5)$$

Using in (4),

$$\therefore \quad \text{Average co-energy density} = \frac{1}{4}\mu_0 H_r^2 \quad ...(6)$$

$$\text{Volume of air gap} = \pi D l g \quad ...(7)$$

Where
D = Average diameter of air gap
l = Axial length of air gap
g = Air gap length
μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ H/m

∴ Total co-energy of the field = Average co-energy density × Volume

$$\therefore \quad W_f' = \frac{1}{4}\mu_0 H_r^2 \times \pi D l g$$

$$= \frac{\pi}{4}\mu_0 \left[\frac{F_r}{g}\right]^2 D l g$$

$$\therefore \quad W_f' = \frac{\mu_0 \pi D l}{4g} F_r^2 \quad ...(8)$$

Using (2) in (8),

$$W_f' = \frac{\mu_0 \pi D l}{4g}[F_1^2 + F_2^2 + 2F_1 F_2 \cos\alpha] \quad ...(9)$$

The torque developed is the partial derivative of the field energy with respect to angle α.

$$\therefore \quad T = +\frac{\partial W_f'}{\partial \alpha}$$

$$= -\frac{\mu_0 \pi D l}{4g} 2F_1 F_2 \sin\alpha$$

$$\therefore \quad \boxed{T = -\frac{\mu_0 \pi D l}{2g} F_1 F_2 \sin\alpha} \quad ...(10)$$

The torque given by the equation (10) is torque per pair of poles as 2 pole machine is considered.

∴ For P pole machine, the torque is given by,

$$\therefore \quad \boxed{T = -\frac{P}{2}\frac{\mu_0 \pi D l}{2g} F_1 F_2 \sin\alpha} \quad ...(11)$$

Key Point : *The equation shows that the torque is proportional to the peak values of stator and rotor m.m.f. waves F_1, F_2 and the sine of the electrical angle α between them.*

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